P3-A-2

Formal Conceptual Blending

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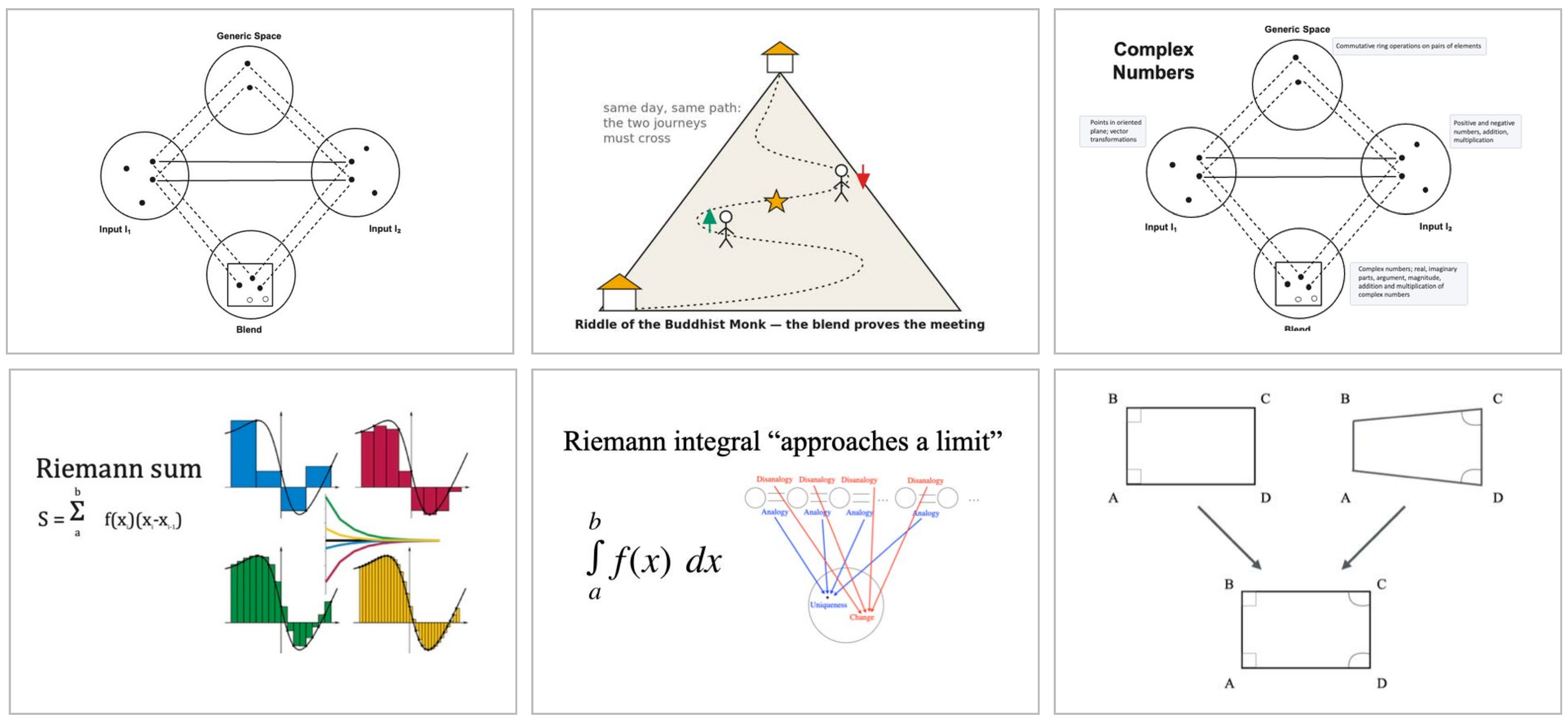
As a fundamental tool for Artificial Mathematical Intelligence and for the fine-tuning of Large Language Models

Blending? What does blending have to do with mathematics?

Conceptual blending creates networks of mental spaces: input mental spaces, a blended mental space, selective projection to the blend, emergent structure in the blend unavailable from an input, and a generic mental space.

Blending is a domain-general engine of human thought — and of mathematical invention. Blending is explicitly part of mathematical practice. Examples include complex numbers, the group of automorphisms of a field extension, and the limit of a Riemann integral. Saccheri, e.g., blended distinct Euclidean configurations: the blend provides a basis for hyperbolic geometry.

The vocabulary of an integration network
input spaces · generic space · cross-space mapping · vital relations · selective projection · compression · emergent structure · the blend



A gallery for the curious (blending.stanford.edu)

Can we formalize the blending engine of the mathematical mind?

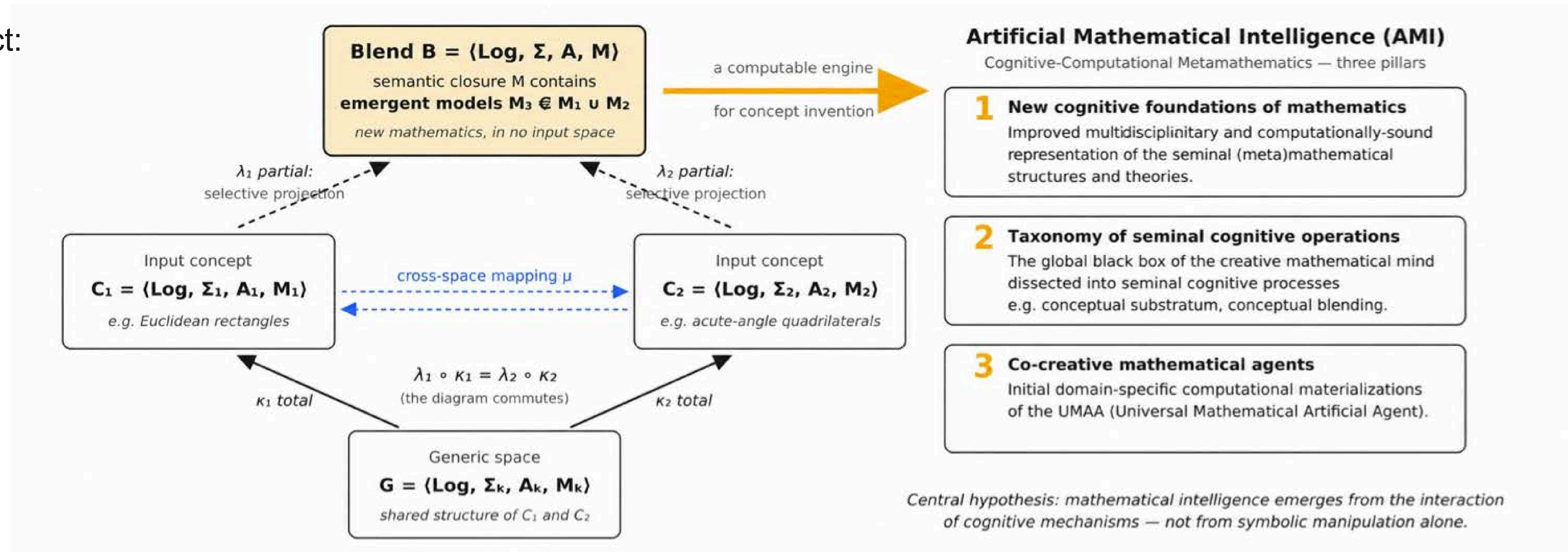
A mathematical concept is a structured, computable object:

$$C = (Log,\ \Sigma,\ A,\ M)$$

- Log — the underlying logical system
- Σ — the signature: the symbols of the concept
- A — the axioms
- M — the semantic closure: all models satisfying A

A blend B of concepts C_1 and C_2 along a generic space G must provide:

- partial cross-space mappings between the inputs
- selective projection into the blend (λ1, λ2)
- **emergent structure: models beyond M1 ∪ M2**
- integration and unpacking of the network



Formal conceptual blending — a V-diagram of total metaphors (κ) completed by partial metaphors (λ) into a blend — is the core generative mechanism of Artificial Mathematical Intelligence.

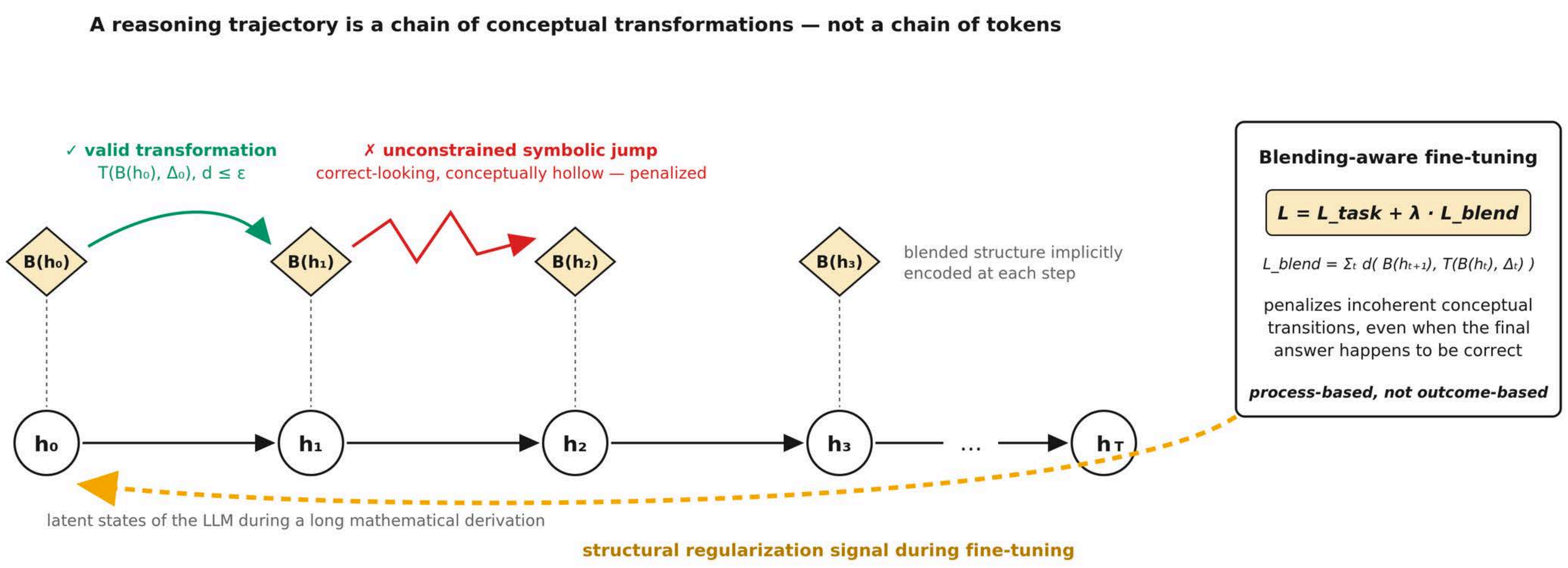
Why do LLMs fail at long-horizon mathematics — and how can blending repair training?

Benchmarks show that frontier LLMs fail frequently in advanced mathematics — not at symbol manipulation, but at sustaining **coherent conceptual structure** across long derivations.

Typical breakdowns:
- lemma invention
- invariant preservation
- reformulation of conjectures
- long-horizon reasoning

The pathology: correct-looking answers supported by internally inconsistent reasoning — ***failures of conceptual integration.***

$$L = L_{task} + \lambda \cdot L_{blend}$$



Blending-aware fine-tuning: a structural regularizer scores each latent transition against a valid transformation of the established blended structure — shifting evaluation from outcomes to reasoning integrity.

Take-home message

Mathematical intelligence is not merely symbolic computation. It emerges from the controlled generation, transformation, and integration of conceptual structures — in humans, and in the artificial systems we should now build.

Main contributions

- A global Taxonomy of Seminal Cognitive Operations for mathematics
- A formal framework for concepts, metaphors, and conceptual blends
- AMI / CCMM as a cognitively grounded paradigm for mathematical AI
- LLM failures reinterpreted as failures of conceptual integration
- Blending-aware fine-tuning and process-based benchmarking

Read more
blending.stanford.edu

Come talk with me about:

1 · Blending — and blending in mathematics
2 · Formal blending for Artificial Mathematical Intelligence
3 · Blending-aware fine-tuning of LLMs

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